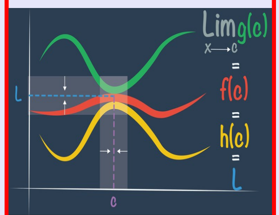
Calculus I

Lecture 18



Feb 19-8:47 AM

Class Quiz 8

Evaluate $\lim\limits_{x\to 0} \dfrac{\sin 2x}{\sin x} = \dfrac{\sin 2(0)}{\sin 0} = \dfrac{\sin 0}{\sin 0} = \dfrac{0}{0}$ I.F.

Method I:

$$\lim_{x\to 0} \frac{\sin 2x}{\sin x} = \lim_{x\to 0} \frac{2\cancel{\sin x}\cos x}{\cancel{\sin x}} = \lim_{x\to 0} 2\cos x = 2\cdot\cos 0 = 2\cdot 1 = \boxed{2}$$

Method II:

$$\lim_{x\to 0} \frac{\sin 2x}{\sin x} = \lim_{x\to 0} \frac{\frac{2\sin 2x}{2x}}{\frac{\sin x}{x}} = \frac{\lim\limits_{x\to 0} \frac{2\sin 2x}{2x}}{\lim\limits_{x\to 0} \frac{\sin x}{x}}$$

$$= 2\cdot \frac{\lim\limits_{x\to 0} \frac{\sin 2x}{2x}}{\lim\limits_{x\to 0} \frac{\sin x}{x}} = 2\cdot\frac{1}{1} = 2\cdot 1 = \boxed{2}$$

Sep 26-7:12 AM

Class Quiz 7

Use $\boxed{m_{\substack{\text{tan. line}\\ \text{at } x=a}} = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}}$ to find

slope of the tan. line to the graph of

$\boxed{f(x)=\frac{1}{x}}$ at $\boxed{x=a}$, $a\neq 0$.

$$m=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\lim_{x\to a}\frac{\frac{1}{x}-\frac{1}{a}}{x-a}=\lim_{x\to a}\frac{\cancel{x}a\cdot\frac{1}{\cancel{x}}-x\cancel{a}\cdot\frac{1}{\cancel{a}}}{xa(x-a)}$$

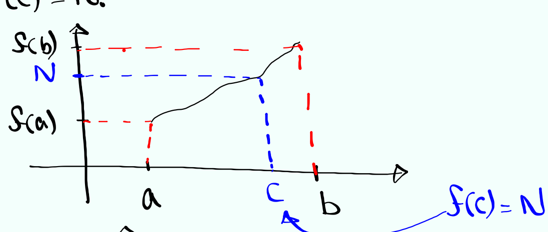
LCD $= xa$

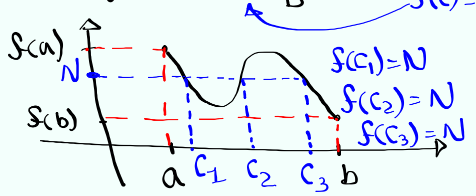
$$=\lim_{x\to a}\frac{a-x}{xa(x-a)}=\lim_{x\to a}\frac{-1}{xa}=\boxed{\frac{-1}{a^2}}$$

Sep 25-8:16 AM

Intermidate Value Theorem:

Suppose $f(x)$ is a continuous function on $[a,b]$, and let N be a number between $f(a)$ & $f(b)$, $f(a)\neq f(b)$, then there is at least a number c on (a,b) such that $f(c)=N$.





Sep 26-7:46 AM

Show $4x^3-6x^2+3x-2=0$ has a Solution between 1 & 2.

$\to f(x)=4x^3-6x^2+3x-2$

$\to a=1$

$\to b=2$

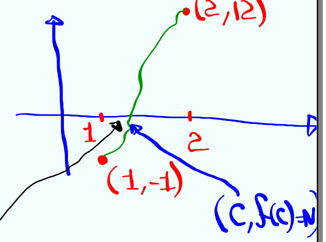
$f(x)$ is a Polynomial Function

It is cont. everywhere.

$f(1)=4(1)^3-6(1)^2+3(1)-2=-1$

$f(2)=4(2)^3-6(2)^2+3(2)-2=12$

$N=0$



$f(a)<N<f(b)$

$-1<0<12$

By I.V.T., there is at least a number on $(1,2)$ Such that $f(c)=0$

Sep 26-7:54 AM

Show $\sqrt[3]{x}=1-x$ has a solution $(0,1)$

$a=0$ $\quad$ $b=1$

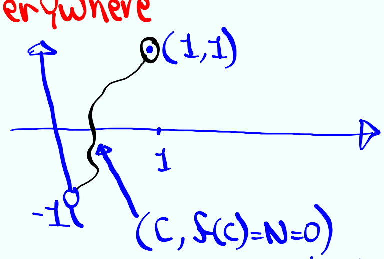
$$\underbrace{\sqrt[3]{x}-1+x}=0$$

$$f(x)=\sqrt[3]{x}-1+x$$

Cont. everywhere

$f(a)=f(0)=-1$

$f(b)=f(1)=1$

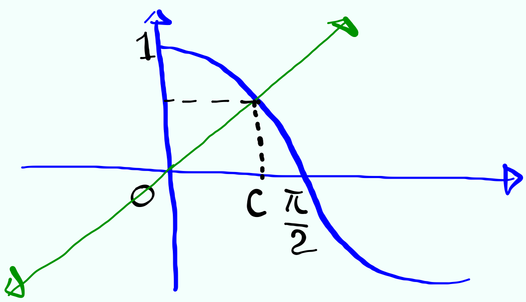


by I.V.T., there is at least a number C on $(0,1)$ such that $f(c)=N$

$f(c)=0$

Sep 26-8:01 AM

Show $\cos x = x$ has a solution in $(0,1)$.



$\underbrace{\cos x - x}$ $= 0$ N

$f(x) = \cos x - x$

Cont. everywhere

$f(0) = \cos 0 - 0 = \boxed{1}$ $\qquad f(1) = \cos 1 - 1 = \boxed{-.5}$

By I.V.T., there is a number c in $(0,1)$ such that $f(c) = 0$

Sep 26-8:06 AM

Find K such that $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ K & \text{if } x = 1 \end{cases}$

is cont. at $x = 1$. To be cont.

$$\lim_{x \to 1} f(x) = f(1)$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2 - x}{x^2 - 1} \quad \frac{0}{0} \text{ I.F.}$$

$$= \lim_{x \to 1} \frac{x\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \lim_{x \to 1} \frac{x}{x+1} = \boxed{\frac{1}{2}}$$

$$\boxed{\frac{1}{2} = K}$$

Sep 26-8:13 AM

Find K such that $f(x)=\begin{cases} \dfrac{2x^2-5x-3}{x-3} & \text{if } x \neq 3 \\ 2K & \text{if } x=3 \end{cases}$

is Cont. at $x=3$.

$f(x)$ is cont. at $x=3$ if $\lim\limits_{x\to 3} f(x) = f(3)$

$7 = 2K$

$\lim\limits_{x\to 3} \dfrac{2x^2-5x-3}{x-3} = \dfrac{0}{0}$ I.F.

$\lim\limits_{x\to 3} \dfrac{(x-3)(2x+1)}{x-3} = \lim\limits_{x\to 3}(2x+1)$

$= 2(3)+1 = 7$

$\boxed{K=\dfrac{7}{2}}$

Sep 26-8:18 AM

Use ε and δ to prove $\lim\limits_{x\to 2} \dfrac{x^2+x-6}{x-2} = 5$

$f(x) = \dfrac{x^2+x-6}{x-2} = \dfrac{(x-2)(x+3)}{x-2}$ $a=2$ $L=5$ ✓

$= x+3$

$\left|\dfrac{x^2+x-6}{x-2} - 5\right| < \varepsilon$ whenever $|x-2|<\delta$

$|x+3-5| < \varepsilon$ " $|x-2|<\delta$

$|x-2|<\varepsilon$ " $|x-2|<\delta$

Pick $\delta = \varepsilon$

Sep 26-8:25 AM

Max Value

Use graphing to show $\boxed{\delta = \sqrt{9+\varepsilon} - 3}$

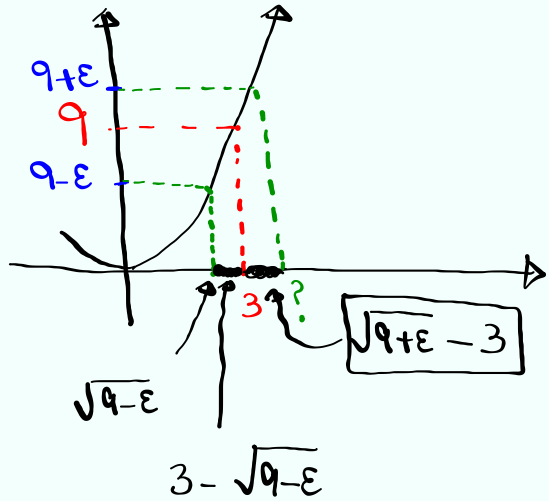
for $\lim\limits_{x \to 3} x^2 = 9.$

$x^2 = 9 + \varepsilon$

$x = \sqrt{9+\varepsilon}$

$x^2 = 9 - \varepsilon$

$x = \sqrt{9-\varepsilon}$



Sep 26-8:29 AM